PRE-SERVICE ELEMENTARY TEACHERS' FRAMING OF MATHEMATICAL DISCUSSIONS AFTER PROBLEM-SOLVING THROUGH MURSION™ SIMULATION

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Research on pre-service teachers' discussion practices has focused on decompositions of practice into subskills, while acknowledging the importance of the role of context, identity, and relationships between interactive moves. We focused on 66 elementary preservice teachers' (PSTs') framing-launching moves in discussions after problem-solving in a Mursion™ custom simulation. PSTs used five moves: gathering information about student processes, focusing on problem features, task and non-task oriented social interactions, and partner talk. Empirical findings of PSTs' intentions and tacit actions coupled with study findings of the diversity in PSTs' framing moves, highlight the complexity of teacher decision making involved in discussion subsills such as framing. We argue that PSTs' framing moves are motivated by an array of intentions including the mathematics aims of discussions.

Keywords: Preservice Teacher Education, Teacher Knowledge.

Inquiry into mathematics teachers' discussion practice has focused on relationships between teacher and student moves (Bishop et al., 2022; Leatham et al., 2015; Stockero et al., 2022). Yet inquiry into preservice teachers' (PSTs') discussion practices highlights discussion subskills within decompositions of practice (Grossman et al., 2009) including planning (Tyminski et al, 2014) and questioning (Moyer & Milewicz, 2002). Shaughnessy et al. (2021) hypothesized that discussion practice contains three areas: framing-launching, orchestrating, and record/representing content (p. 455). Framing-launching a discussion within decomposition of practice involves teacher moves including focusing attention on mathematics but differs from launching a task in that a goal of the launch is to prepare students to share ideas about strategies rather than prepare students for problem-solving. Mathematics teacher educators need a better understanding of specific moves teachers use for framing-launching mathematics discussions to develop pedagogical learning trajectories for teacher discussion practice needed to inform development activities for PSTs. We report on PSTs' framing-launching (Shaughnessy et al., 2021, p. 455) of discussion after problem-solving, which we will refer to as framing. We argue that PSTs' framing moves are motivated by factors that include but are not limited to mathematical aims for discussions.

Drawing from Steffe and D'Ambrosio's (1995) description of constructivist teaching, we view learning to teach as dynamic testing of practice. Such testing involves the use of tacit and explicit theories involved in educative interactions such as social and socio-mathematical norms (Yackel & Cobb, 1996). Generalizations of normative behavior can reproduce societal inequities, yet PST and student interactions can also challenge generalizations. PSTs' experience with unexpected student responses, can produce disequilibrium in understandings of norms. PSTs

interact as model builders (Ulrich et al., 2014), creating practices by informal tacit testing of utility and productivity of interactive moves in mathematics discussions with students.

Research into PSTs' preparation for (and engagement in) discussions describes intentions (e.g. Tyminski et al., 2014) and moves (e.g. Shaughnessy & Boerst, 2018) PSTs use. PSTs design discussions to provide students with opportunities to share, learn from sharing, and/or compare strategies (Tyminski et al., 2014). PSTs also use intuitive skills to elicit students' problem-solving process or probing students' understanding about the problem (Shaughnessy & Boerst, 2018). Many PSTs launch interactions with students by asking what they did when solving the problem, "What was your first step when you solved the problem?" (Shaughnessy & Boerst, 2018, p. 52). Some PSTs probe to gather evidence of what students noticed about task features or understood about their problem solving process by asking, "So can you kind of tell me what this row that you wrote stands for or what that number is?" (p. 46). PSTs may also utilize other moves that support learning about students' thinking, such as encouraging students to write or facing students when asking a question (Shaughnessy & Boerst, 2018)

In-service teachers select student work for discussion using the mathematics potential of the work, but also historical patterns of student participation and mathematical understanding (Dunning, 2022). While decompositions of practice (Grossman et al., 2009) play an important role in developing PSTs' practice (Jacobs & Spangler, 2017), discussion is complex and contextual, it involves teachers' interactive moves influenced by factors including teachers' view of learning (Simon, 2008); teachers' identities (Drake, 2006), views of learners' participation (Dunning, 2022), learners' needs (Sztjan, 2003), learners' identities (Rubel et al., 2022), and learners' lived experiences (Van Es et al., 2022). These findings taken together suggest further inquiry into PSTs' subskills of discussion practice (e.g. Tyminski et al., 2014; Shaughnessy et al., 2021) using frameworks for analyzing interactions (e.g. Bishop et al., 2022) to describe PSTs' framing of mathematical discussions after problem-solving.

Studies of teacher discussion practice have focused on teacher moves in relation to students' mathematics. For example, Leatham et al. (2015) defined mathematically significant pedagogical opportunities to build on student thinking (MOSTs) as a tool to explore teachers' use of students' mathematics in discussions. Bishop et al., (2020) described three categories of teacher moves: confirm/correct, probe/publicize, engage, where confirm/correct moves "did not use students' mathematical contributions" (p. 15). Instead, teachers shared their thinking, acknowledged or evaluated student responses. Probe/publicize moves used probing questions to understand student thinking and revoicing to highlight student responses. Engaging moves directed students to engage with others' ideas by asking students to restate, reason about, or apply another students' approach. Bishop et al. (2022) also classified student contributions: participate (sharing of facts or procedures), explain (sharing strategies without justification), and "substantive reasoning" (p. 15). Less clear was how teachers developed discussion practices.

To unpack the development of discussion practices, we report findings from analysis of data from a larger study of PSTs' moves to facilitate discussions with students after problems solving in the context of an online virtual custom simulation of discussion. Based on coding of 66 PSTs' framing moves in a discussion after problem solving we identified five framing moves: (a) gathering information about student processes; (b) focusing on features of the problem; (c) task-oriented social interactions, (d) non-task oriented social interactions; and (e) partner talks. We argue that these initial moves were motivated by factors beyond mathematics.

Use of Mixed-Reality Simulations (MRSs) has increased in teacher education (REF). MRS including human-in-the loop Mursion^{TM} simulations encompasses real and virtual environments

(Gundel et al., 2019). MRSs provide a low-risk environment (Dieker et al., 2014; Piro & O'Callaghan, 2019) for PSTs to develop teaching related theory, practice (e.g., Hudson et al., 2019), and beliefs (e.g., Bautista & Boone, 2015; Gundel et al., 2019). This study uses a Mursion™classroom custom simulation (Grant & Ferguson, 2021) as the context for PSTs to facilitate discussions of mathematics after problem-solving. Prior findings using this simulation include PSTs reports that after facilitating a mathematics discussion in the simulation, they felt more prepared, confident, and their teaching anxiety reduced (Grant & Ferguson, 2021). Our findings extend literature on MRSs' impact from efficacy to describing practice.

Methods

Participants were 23 PSTs in elementary mathematical method courses (fall 2018, spring 2019, fall 2019) at a Mid-Atlantic university and 43 PSTs in elementary methods courses (spring 2021) at a Midwestern university. Analysis of PSTs' framing of discussions after problem-solving is part of a larger study to describe PSTs' development of discussions after problem-solving.

In this study, we analyzed PSTs' framing moves in a Mursion™ classroom simulation. Mursion is a real-time software that creates realistic and interactive environment by blending artificial intelligence and trained simulation specialists who control the speech and movements of avatars (virtual students) in a virtual environment during the simulation (Dieker et al., 2014; Grant & Ferguson, 2021). PSTs sit in front of a screen, with a camera and microphone (Figure 5). Five avatars appear on screen, sitting behind a desk with name cards on the desk (e.g., Savannah, Ethan, Dev, Eva, Jasmin, Harrison, etc.). Avatars' actions and utterances are controlled by a simulation specialist (off screen) who can see and hear the PSTs (Figure 6). Avatars respond as elementary students providing answers, explanations, and asking questions. Further, the avatars reflect various physical and social student characteristics such as hair color, phenotype, quick to volunteer, and talkative through their actions or engagement. While the PSTs were interacting with avatars, we refer to these avatars as students, to highlight the realistic nature of the custom simulation for the PSTs.

Figure 5. Mursion™ Simulation Room

Figure 6. Avatars in Virtual Environment

Study Design

Mursion™ simulation provides an opportunity for PSTs to practice core practices (Jacobs & Spangler, 2017) of teaching. In a custom simulation (Grant & Ferguson, 2021), PSTs had opportunities to orchestrate a mathematical discussion about student's solutions to a problem solving task. In the elementary mathematics method courses, the simulation was used as a microteaching environment (Grossman et al., 2009; Ledger et al., 2019). PSTs were directed to elicit students' reasoning and justification, and to help students make meaningful connections

across strategies generated through problem-solving to strengthen mathematical understanding (Graeber, 1999; Jacobs & Spangler, 2017; Smith & Stein, 2018). The mathematical discussion focused on student solutions of "Harry the Dog Problem” (Figure 7).

Harry, the dog, is standing in a line of 23 dogs, waiting to be washed. He is hot and impatient.

Each time a dog is washed and Becky, the groomer, turns to let the dog into the cool doggie run, Harry sneaks ahead of two dogs. How many dogs will be washed before Harry?

Figure 7. Harry the Dog Problem

We provided 6 solutions to "Harry the Dog Problem”, and told the PSTs that the solutions were generated by the students during problem-solving (Table 2). PSTs did not know which avatar had which solution. The PSTs were given five to eight minutes to engage the avatars in the Mursion™ simulation with the goal of facilitating discussion after problem-solving to generate mathematical sense-making. We expected PSTs to ask questions to elicit students' thinking by encouraging them to use good mathematical explanations to make sense of problem solutions. PSTs were cautioned not to provide any hints or verify students' answers. Each PST's simulation was video recorded, and the de-identified transcripts of the recordings constitutes the data for this study.

Table 2: Students’ Solutions Provided to PSTs Before Facilitating the Discussion

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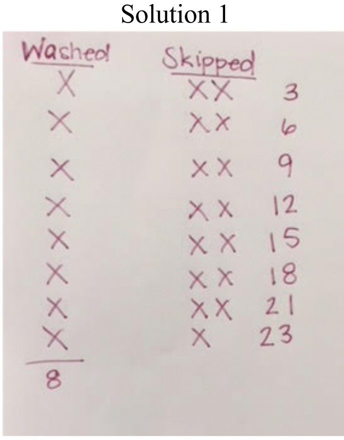
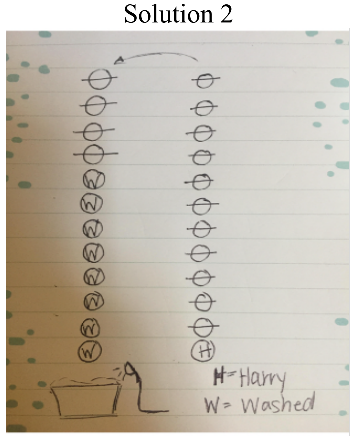
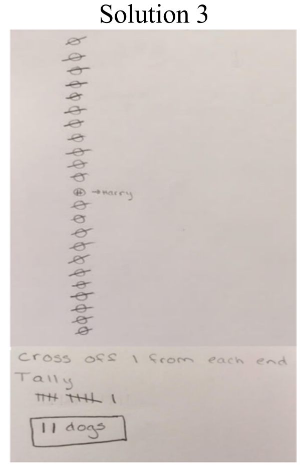
<tr><td>
</td><td>
</td><td>
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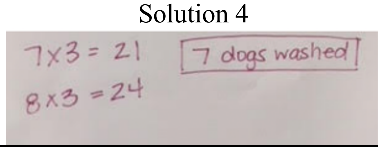
<tr><td>Solution 4</td><td>Solution 5</td><td>Solution 6</td></tr>
<tr><td>
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	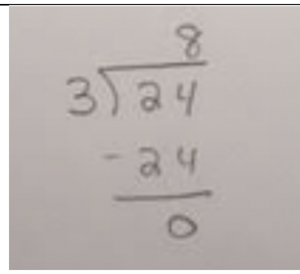	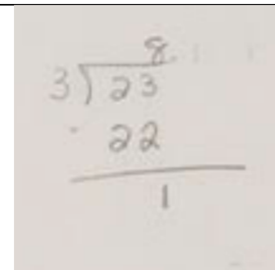

Coding and Analysis

The objective of the coding process was to describe how PSTs framed discussion after problem-solving. We coded the initial discussion interactions between PSTs and the students, drawing on Shaughnessy and Boerst's (2018) classification of PSTs' moves of eliciting students' problem-solving process and probing students' understanding about the problem. We also identified new moves PSTs used to support learning about students' thinking (Table 3). All PSTs' framing moves were coded by two members of the research team to guarantee consistency throughout the data (Braun & Clarke, 2006).

Table 3: PSTs' Discussion Moves in Mathematical Discussion

Framing Moves to Initiate Discussion	Description
Gathering students answers or strategies (see elicit students' thinking, Shaughnessy & Boerst, 2018)	PSTs initiate the discussion by posing questions to learn students' answers or strategies for the problem solving process (e.g., Can you tell me how you solved the problem?)
Focusing on features of the problem (see probing students' understanding; Shaughnessy & Boerst, 2018)	PSTs initiate the discussion by posing questions to learn students' understanding of key mathematical features of the problem or draw students' attention to key points of the problem (e.g., How many dogs are in the line?).
Other Moves (Social Moves)	**Description**
Task oriented social interactions (see also other moves to support student thinking; Shaughnessy & Boerst, 2018)	PSTs initiate the discussion by engaging in task oriented social interactions. They pose task oriented questions to elicit students' emotions or opinions regarding the task or problem solving process to prepare them for the discussion.(e.g., Do you remember the Harry the Dog problem?).
Non-task oriented social interactions (see other moves to support student thinking; Shaughnessy & Boerst, 2018)	PSTs initiate the discussion by engaging in non-task oriented social interaction to greet or gauge students' energy before digging into the problem solving process (e.g., How are you today?).
Partner Talk (see other moves to support student thinking; Shaughnessy & Boerst, 2018)	PSTs initiate partner talk, directing students to engage in a conversation with peers to talk about each other's process and answer (e.g., I want you to turn to your partner and talk about how you solved the problem?).

Findings

Overall, majority of the PSTs initiated the discussion after problem solving by gathering student answers or strategies (Table 4). Fifty seven of 66 PSTs (86%) framed discussions by

gathering student answers or strategies, a framing move that involves the PSTs selecting *a volunteer or one student* to share. The student responded by providing an answer (e.g. eight) or a descriptor of their approach (e.g. multiplied, created a chart). For example, PST1 chose Savannah to begin the discussion by inviting her to share the problem solving process.

PST1: Hi guys! So, we are going to talk about the Harry the dog problem again. So, do you guys want to walk me through, kind of what your thought processes were when you were doing the problem? Savannah, do you want to go ahead and start?

Savannah: Okay! So, I did multiplication. And I did seven times three is twenty one and eight times three is twenty four but we only have twenty three dogs. So, I got seven.

Alternatively, some PSTs who gathered students' answers or strategies to initiate the discussion *solicited volunteers* to share their solution. Specifically, 30 of 57 PSTs (53%) who used the gathering student answers or strategies moves invited a volunteer rather than selecting a specific student. For example, PST2 affirmed Harrison's request that the class go over the problem and again asked for a volunteer.

PST2: Let's ask the question! Who can give me a solution to the problem?

Harrison: It was kind of hard. Yeah, I don't know. Yeah, it was a pretty tough one. Are we gonna go over it at all?

PST2: Yes. Can somebody give me what they got? And we'll talk about it.

Savannah: Oh, Well, I yeah, I think it was five.

Seven of 66 PSTs (11%) drew students' attention to problem features. PST3 selected a student and invited him to share key mathematical features of the problem, an important move to support problem solving discussions.

PST3: … So the first thing we're going to look at is who can raise their hand and tell me what keywords that you found when going into this that you saw immediately that stood out to you that would help you go about answering this question. Harrison

Harrison: Oh, well, I mean, just the 23 stood out to me right away, just because I mean, you know, I know it's gonna be a math problem. And you know, just set the scene of like, how many dogs were in the line ahead of Harry.

Some PSTs who attempted to draw students' attention to problem features (see Table 3), solicited volunteers rather than selecting a student to initiate the discussion. PSTs 4 and 5 used the number of dogs in line and invited a volunteer to discuss a key feature of the problem.

PST4: Okay, class, let's begin with going over the problem. We want to get some information. So how many dogs are in line?

PST5: So, we've been working on the Harry the dog problem, it's a math problem, but a dog and he's in line to get washed with a bunch of other dogs, right? And it says that Harry is the 23rd dog in line, or is he the 24th dog? Now, this question set up

a little bit odd and it's a different interpretation. So I'm going to see how many of you, go ahead and raise your hand, has have Harry is the 24th dog in line?

Table 4: A Breakdown of the Discussion Moves in Mathematical Discussion Used by PSTs

	Framing moves		**Other moves (Social Moves)**		
	Gathering student answers or strategies n (%)	**Focusing on features of the problem n (%)**	**Task oriented n (%)**	**Non-task oriented n (%)**	**Partner talk n (%)**
Mid-Atlantic University (n=23)	21 (91%)	2 (9%)	1 (4%)	7 (30%)	---
	Selecting a volunteer 12 (57%)	**Selecting a volunteer** 2 (100%)			
Midwestern University (n=43)	36 (84%)	5 (12%)	10 (23%)	17 (40%)	5 (12%)
	Selecting a volunteer 18 (50%)	**Selecting a volunteer** 4 (80%)			

Other Moves

Twenty four of the 66 PSTs (36%) engaged in non-task oriented social interactions before framing the mathematical discussion. These teachers made an effort to greet or gauge student energy, readiness for discussion. For example, PST6 began with an emotionally focused question about feelings

PST6: How are we doing today?

Jasmine: Hi, really good. I like your scarf.

PST6: Thank you. I like your shirt.

Jasmine: Thank you.

PST6: So does anyone want to start off with their problem? Anyone want to explain to me what they did for Harry the dog?

Eleven of the 66 PSTs (17%) engaged in task oriented social interactions before framing the mathematical discussion. These PSTs concentrated on initiating the discussion by posing more task-oriented questions to elicit avatars' emotions or opinions regarding the task or solving process, and prepared them for the discussion. For example, PST7 began with a mathematically focused question about students' opinions as a social interaction for readiness of the discussion, i.e., *"Good morning class! Today, we are going to talk about the Harry the dog problem. Do you guys remember that?"*. These PSTs set the stage for student mathematics engagement by first initiating non-task or task-related social interactions attending to students as humans.

Five of 43 PSTs (12%) at the Midwestern University invited students to partner talk to discuss their problem-solving process and answers before initiating a whole class mathematics

discussion. PST8 let the students engage in a small group mathematical conversation for readiness of whole class discussion.

PST8: We are going to talk again, I know you might have already once, but I want to talk again about the Harry the dog problem. So, in the problem, it says, (reading the problem). I saw all of you do your problems and how you solved it. Could you partner talk really quick and talk about how you solved it and then we will meet together and about like 10 seconds, take 10 quick seconds. How did you solve it? Can you talk to your partner?

Some PSTs who used social interactions before framing the mathematical discussion (8 PSTs at Mid-Atlantic university, 32 PSTs at Midwestern University) used at least two social moves prior to initiating discussion of mathematics. One of the eight PSTs (13%) at the Mid-Atlantic university and eight of 32 PSTs (25%) at the Midwestern University engaged in at least two different social interactions before framing the mathematical discussion (e.g., using a non-task oriented social move with a task oriented social interaction; using a non-task oriented social move with partner talk).

Discussion

PSTs discussion practice has been studied using decompositions of practice (Grossman et al., 2009). As anticipated, PSTs at both universities typically framed discussion after problem-solving by gathering information about student processes (Shaughnessy & Boerst, 2018). PSTs' framing moves focused on gathering information about student processes, focusing on features of the problem, and social interactions, rather than focusing on mathematics reasoning of a selected work sample.

At first glance, PSTs' initial move may seem inefficient in that with student solutions available, PSTs could choose to begin with a student work based on mathematical potential (Leatham et al., 2015). Yet Dunning (2022) identified teachers' use of factors including student historical patterns of participation and mathematics understanding in selecting student work for mathematics discussions. Additionally, Tyminski et al. (2014) noted that PSTs' intention for discussions often focused on more general aims such as strategy sharing to enable students to learn from others' work. The complexity involved in teacher decisions associated with discussion subskills including framing-launching (Shaughnessy et al, 2019) suggests additional study of "interactive detail" and "teacher voice" is needed (Jacobs & Spangler, 2017, p. 704) to contribute to our understanding of PSTs' modeling of students as mathematics learners. Additionally, PSTs' assumptions about learning (Simon, 2008) coupled with details of interactions are needed to discern how PSTs are developing discussion practices.

PSTs' views of students' mathematics develop through interactions (Steffe & D'Ambrosio, 1995). PSTs as model builders take interactive moves (e.g., Bishop, 2022) that shape mathematical discussions (Ulrich et al., 2014). Interactive moves are influenced by factors beyond mathematics including teacher identities (Drake, 2006), learner identities (Rubel et al., 2022), and perceptions of learner participation (Dunning, 2022). In our study, PSTs' relied on hearing students' processes to frame discussions, yet framing moves also attended to students as humans and mathematicians. Taken together, the research findings support the argument that PSTs' framing moves are motivated by intentions beyond mathematical aims of discussions. Additional exploration of factors such as student identities that motivate PSTs' framing moves is needed to model PSTs' discussion practice.

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